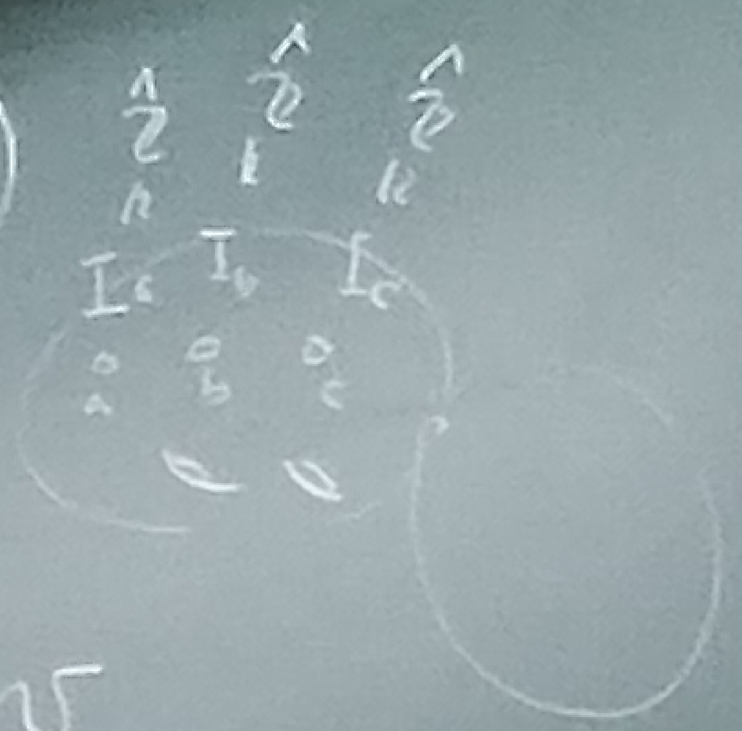
$\mathcal{G}_e := \mathcal{B}$(cuspidal inertia ass to e)

$\mathcal{B}$(Spec $k[[t]][\frac{1}{t}]$))



(iv) $\forall v \in \mathcal{V}(\mathcal{G})$, $\forall e \in \mathcal{E}(\mathcal{G})$ abuts to v

, $\forall b$: branch of e induced by

$b_{e\to v} : \mathcal{G}_e \to \mathcal{G}_v \overset{\text{def}}{:=} \mathcal{B}(\ ) \to \mathcal{B}(\ )$ " $I \hookrightarrow \pi_1(\ )$ "

(3) semi-graph of anabelioids

of PSC-type $\mathcal{G}$

(covering of $\mathcal{G}$)

$\exists \mathcal{B}(\mathcal{G})$: category

obj : $\{A_v, \phi_e\}_{v,e}$ $\quad \exists A_v \in \mathrm{Obj}(\mathcal{G}_v)$

$\phi_e : b^*_{e\to v_1}(A_{v_1}) \xrightarrow[\text{isom}]{\sim} b^*_{e\to v_2}(A_{v_2})$

mor : ______

$\Rightarrow$ conn anabelioid

$\rightsquigarrow \Pi_{\mathcal{G}}$: fund gp of $\mathcal{G}$

ex

cuspidal inertia ass to e) $\hat{I}$ $\hat{I}$ $\hat{I}$

$I_e \; I_e \; I_e$

(Spec $k[[t]][\frac{1}{t}]$))

$\forall e \in \mathcal{E}(\mathbb{G})$ abuts to v

...ch of e induced by

$\stackrel{\text{def}}{=} \beta(\;) \to \beta(\;)$ " $I \hookrightarrow \pi_1(\;)$ "

...oids [cf. theory of admissible coverings]

...SC-type $\mathcal{G}$

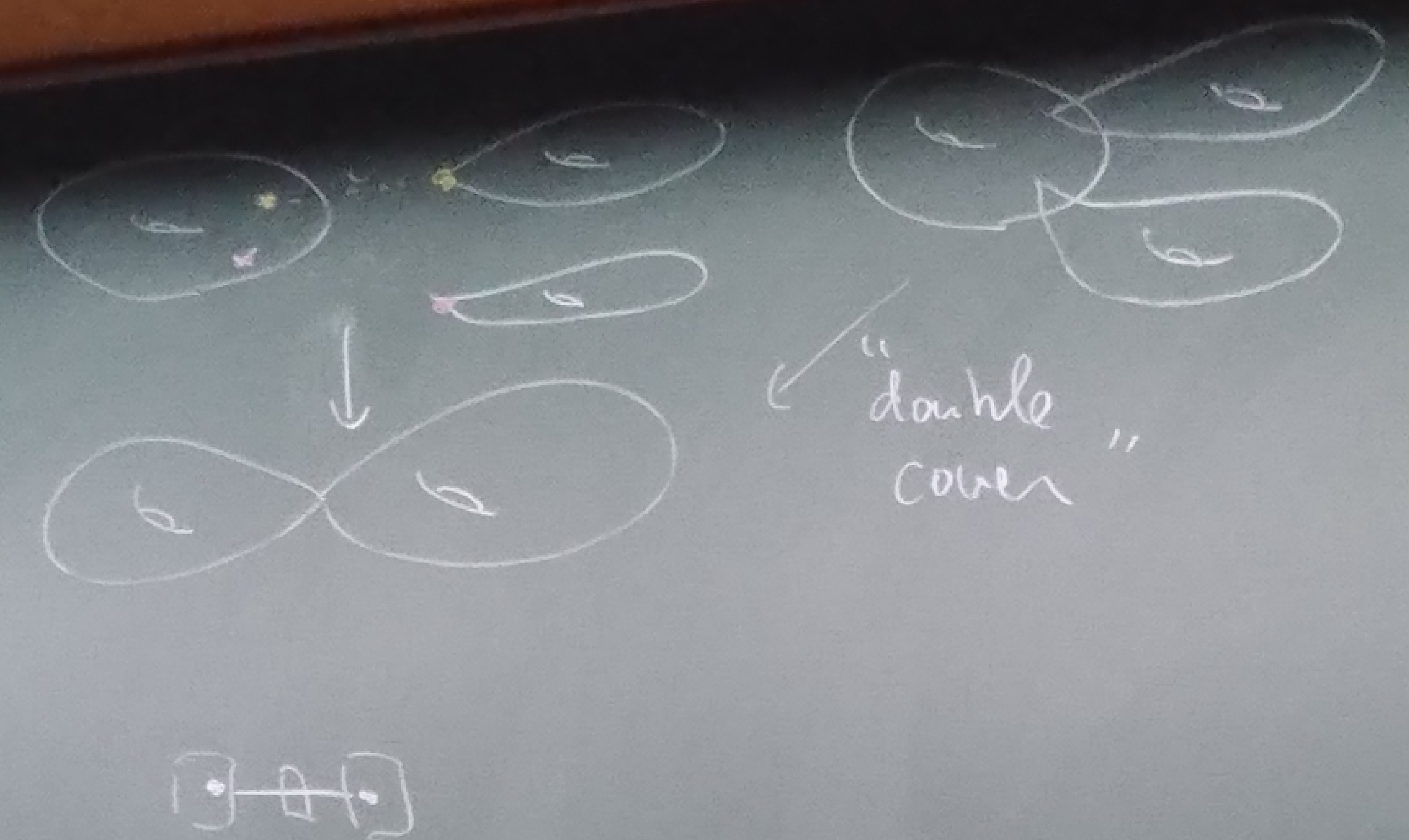
Reference
- Mzk compactification of Hurwitz sch
- Mzk profinite Grothendieck conj

obj: $\{A_v, \phi_e\}_{v,e}$ $\exists A_v \in \mathrm{obj}(\mathcal{G}_v)$

$\phi_e : b^*_{e\to v_1}(A_{v_1}) \xrightarrow[\text{isom}]{\sim} b^*_{e\to v_2}(A_{v_2})$

...mor:

ex



$\tilde{v}$ $\tilde{\mathcal{G}}$ $\tilde{e}$

$v \in \mathcal{G} \ni e$

(4) semi-graph of anabelioids of PSC-type $\mathcal{G}$

$\forall v \in V(\mathbb{G}) \rightsquigarrow \Pi_v \subseteq \Pi_{\mathcal{G}}$ (up to $\Pi_{\mathcal{G}}$-inner)

$\Rightarrow$ verticial subgroup

$\forall e \in \mathcal{E}(\mathbb{G}) \rightsquigarrow \Pi_e \subseteq \Pi_{\mathcal{G}}$ (up to $\Pi_{\mathcal{G}}$-inner)

$\Rightarrow$ edge-like subgroup

(ii)

anabe...

(iii)

Prop 1

$$\begin{array}{ccc} \tilde{v} & \tilde{\mathcal{G}} & \tilde{e} \\ & \downarrow & \\ v \in & \mathcal{G} & \ni e \end{array}$$

Prop 1 $\mathcal{G}$: semi-graph of ______ of PSC-type

(1) $\Pi_{\mathcal{G}}, \Pi_v$: slim

($\forall H$: open subgp, $Z(H) = \{1\}$)

(2) $N_{\Pi_{\mathcal{G}}}(\Pi_v) = \Pi_v$, $N_{\Pi_{\mathcal{G}}}(\Pi_e) = \Pi_e$ (normally terminal)

commensurably terminal

$\Downarrow$

normally terminal

Rmk [NSW]

k : NF $\mathfrak{p}$: non-arch prime of k

(1) G_k, $G_{k_\mathfrak{p}}$: slim

(2) $N_{G_k}(G_{k_\mathfrak{p}}) = G_{k_\mathfrak{p}}$

⟨ Combinatorial (version of) Grothendieck conjecture ⟩

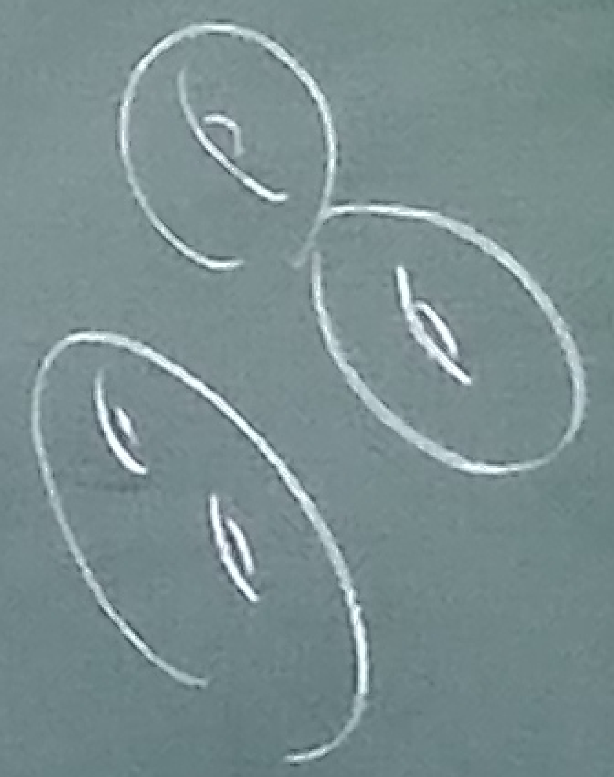
$\mathcal{G}$ semi-graph of ——— of PSC-type

I : prof gp

$\rho : I \longrightarrow \mathrm{Out}(\Pi_{\mathcal{G}})$ cont hom satisfying $^{\exists}$condition

$\Rightarrow$ $^{\forall}\phi \in Z_{\mathrm{Out}(\Pi_{\mathcal{G}})}(\mathrm{Im}(\rho))$ is graphic

i.e., arises from $^{\exists}$ [illegible] $\in \mathrm{Aut}(\mathcal{G})$



$\forall \rho \in \mathcal{E}(\mathcal{G})$

Rmk (orig

k : fiel

X/k : hy

ρ : G_k

$\Rightarrow$ $\forall \phi \in$

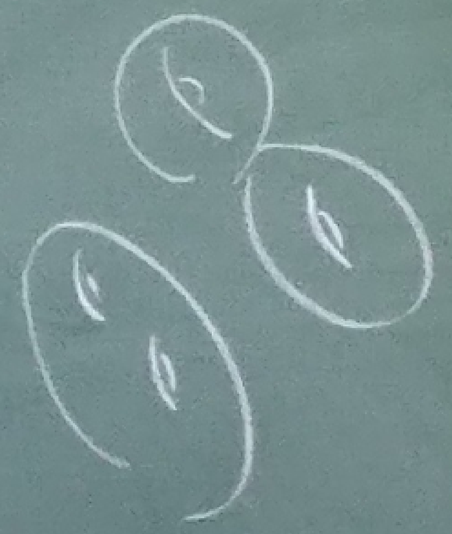
…eck conjecture⟩

— of IPSC-type

… from satisfying $^{\exists}$ condition

graphic

$\mathrm{Aut}(\mathcal{G})$



[NSW]

(1)

(2)

(2)

Rmk (original Grothendieck conjecture)

k : field satisfying $^{\exists}$ condition

X/k : hyperbolic curve

$\rho : G_k \longrightarrow \mathrm{Out}(\pi_1(\overline{X}))$ outer Gal rep

$\Rightarrow \forall \phi \in Z_{\mathrm{Out}}(\mathrm{Im}(\rho))$

arises from $^{\exists}$ [illegible] $\in \mathrm{Aut}_k(X)$

Thm (Mochizuki)

For $\phi \in Z_{\mathrm{Out}}(\mathrm{Im}(\rho))$

- ρ : IPSC-type
- ϕ : group-theoretically cuspidal

induces a bijection between the set of cuspidal inertia subgps

$\Rightarrow$ ϕ : graphic (i.e., CombGC holds)

$$1 \to \underset{\mathrm{Ker}}{\overset{}{\Pi_{\mathcal{G}}}} \to \pi_1(X^{\log}) \to \pi_1(S^{\log}) \to 1$$

IPSC-type

$\mathcal{G}$: semi-graph of anabelioids

$\rho : I \to \mathrm{Out}(\Pi_{\mathcal{G}})$: IPSC-type

$\overset{\text{def}}{\iff}$ $\exists\ X^{\log} \to S^{\log} = (\mathrm{Spec}\, L, \mathbb{N})$

L : alg closed field ch$=0$

$X^{\log} \to S^{\log}$: Stable log curve

$$\begin{tikzcd} {} \arrow[r] \arrow[d] \arrow[dr, phantom, "\square"] & \overline{\mathcal{M}}_{g,r+1}^{\log} \arrow[d] \\ S^{\log} \arrow[r] & \overline{\mathcal{M}}_{g,r}^{\log} \end{tikzcd}$$

s.t. "ρ" $\cong$ "$\pi_1(S^{\log}) \to \mathrm{Out}(\mathrm{Ker})$"

Rmk · NN-type ($\Leftarrow$ IPSC)

$\uparrow$ "purely gp-theoretic" condition

Reference:

Hoshi-Mochizuki

On the combinatorial anabelian geometry of nodally nondegenerate ——

· $\exists$ CombGC for NN-type representations

Than

$= \mathrm{IP}^{S}_{C}\mathrm{C})$

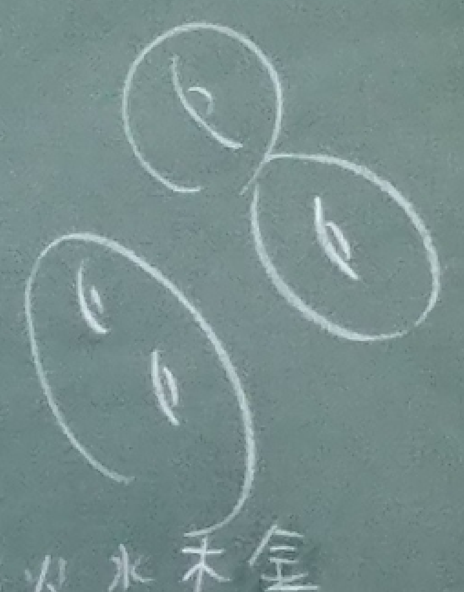
15:15 ~ start again

"...heoretic" condition

N-type representations

graphic

$ut(\mathcal{G})$



2月 月 火 水 木 金

1 2 3 4 5

8 9 10 11 12

15 16 17 18 19

22 23 24 25 26

29

Next month 2/16 13:00~ K. Nakamura

§3 In §3, §4, we assume $k = \bar{k}$, $ch(k) = 0$

$\mathrm{Out}^{FC}(\Pi_2) \to \mathrm{Out}^{FC}(\Pi_1)$

<u>lem 1</u> X : arbitrary hyperbolic curve $/k$

Then
$$\begin{tikzcd} & \mathrm{Out}^{FC}(\Pi_1) \\ \mathrm{Out}^{FC}(\Pi_2) \arrow[ur, "p_1"] \arrow[dr, "p_2"'] & \\ & \mathrm{Out}^{FC}(\Pi_1) \end{tikzcd}$$
coincide

$$\begin{tikzcd} & X_1 \\ X_2 \arrow[ur, "pr_1"] \arrow[dr, "pr_2"'] & \\ & X_1 \end{tikzcd}$$

$(\rightsquigarrow \mathrm{Out}^{FC}(\Pi_{n+1}) \to \mathrm{Out}^{FC}(\Pi_n)$: natural $)$

$\alpha \in \mathrm{Aut}^{FC}(\Pi_2)$

$$\begin{tikzcd} & \Pi_1 \curvearrowleft \alpha_1 \\ \alpha \curvearrowright \Pi_2 \arrow[ur, two heads, "p_1"] \arrow[dr, two heads, "p_2"'] & \\ & \Pi_1 \curvearrowleft \alpha_2 \end{tikzcd}$$

it suffices to show that

"$\alpha_2 = \mathrm{Inn} \circ \alpha_1$"

Let $\mathbb{I} \subseteq \mathbb{D} \subseteq \Pi_2$ associated to diagonal divisor

$\mathbb{I}$: inertia subgp

$\mathbb{D}$: decomp subgp

Thus, w...

$\mathbb{D}$

Π_1